Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name : Group Theory

Subject Code : 4SC05GTC1 **Branch : B. Sc. (Mathematics)**

Semester : 5 Date : 4 / 12 / 2015 Time : 2:30 To 5:30 Marks : 70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) If $a^2 = e$ for each element a of a group G then G is abelian. **[2]**

b) Define: Index of H in G. **[2]**

c) Show that $f \in S_{15}$, where **[2]**

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 7 & 10 & 13 & 4 & 5 & 12 & 3 & 15 & 2 & 9 & 11 & 6 & 1 & 14 & 8 \end{pmatrix}$$

is an odd permutation.

d) If H and K are finite subgroups of a group G and $O(H)$ & $O(K)$ are relatively prime integers then show that $H \cap K = \{e\}$. **[2]**

e) Is $(Z_5, +_5)$ a cyclic group? Justify your answer. **[2]**

f) Show that the mapping $\phi: (R, +) \to (R_+, \cdot)$, $\phi(x) = e^x$ is an isomorphism. **[2]**

g) If $(ab)^2 = a^2b^2, \forall a, b \in G$,show that G is commutative. **[2]**

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) State and prove Cayley's theorem. **[7]**

b) If $G = \{e, a, a^2, \dots, a^{23} / a^{24} = e\}$ is a cyclic group of order 24 and $H =< a^6 >$, prepare the group table for the quotient group G/H. Using this table, answer the followings: **[7]**

(1) Find the inverse of Ha^4 in G/H.

(2) Solve the equation $y(Ha) = Ha^5$ in G/H.

Q-3 **Attempt all questions** **(14)**

a) If $\phi: G \to G'$ is an onto homomorphism with kernel K , show that $G/K \cong G'$. **[7]**

b) Show that $O(A_n) = \frac{n!}{2}$ and hence show that A_n is normal in $S_n; n \geq 2$. **[7]**

Q-4 **Attempt all questions** **(14)**

a) Show that a group G is commutative if $(ab)^i = a^i b^i,\ \ a, b \in G$ for any three consecutive integers. **[5]**

	b)	Show that any two infinite cyclic groups are isomorphic.	**[5]**
	c)	Show that the intersection of two subgroups of a group is also a subgroup.	**[4]**
Q-5		**Attempt all questions**	**(14)**
	a)	For a subgroup H of G and for $a, b \in G$. Show that $Ha = Hb \iff ab^{-1} \in H$.	**[5]**
	b)	Show that a cyclic group is a commutative group.	**[5]**
	c)	For a given element $a \in G$, prove that $N(a) = \{x \in G/xa = ax\}$ is a subgroup of G.	**[4]**
Q-6		**Attempt all questions**	**(14)**
	a)	State and prove Lagrange's theorem.	**[7]**
	b)	Show that a subgroup of cyclic group is cyclic.	**[7]**
Q-7		**Attempt all questions**	**(14)**
	a)	Obtain all subgroups of $(Z_{18}, +_{18})$ and hence prepare its lattice diagram.	**[7]**
	b)	If $\phi: (G, \cdot) \to (G', *)$ is an onto homomorphism with kernel K then show that $G/K \cong G'$.	**[7]**
Q-8		**Attempt all questions**	**(14)**
	a)	A homomorphism $\phi: G \to G'$ is one-one if and only if $K\phi = \{e\}$.	**[5]**
	b)	A subgroup H of G is a normal subgroup of G if and only if $aHa^{-1} \subset H$ for each $a \in G$.	**[5]**
	c)	If $a \in G$ is of order n then $a^m = e$ for some integer m if and only if n/m.	**[4]**